



RESEARCH-BASED RESEARCH-PROVEN

- Engineered for Learning
- Field Tested and Iteratively Improved
- Verified Success

Everyday Mathematics®

How Children Learn.

GRADES
K-6

everydaymath.com

How Children Learn

At McGraw-Hill Education, we know that behind each student success story is a team of great teachers and administrators who set high expectations for themselves and their students. That's why we set the same high expectations for *Everyday Mathematics*.

That means that when you implement *Everyday Mathematics*, you can be confident that your children's mathematics instruction will be grounded in an extensive body of research into how children learn.

It also means that your students' curriculum will have been subjected to more scrutiny by more researchers than any other program available, a fact that has been verified by a study of the National Academy of Sciences (NRC 2004).

This research points squarely in the same direction: Children who use *Everyday Mathematics* develop deeper conceptual understanding and greater depth of knowledge, and they enjoy learning math a lot more than children who use other math programs. It's how children learn.

everydaymath.com

Research-Based and Research-Proven

Each edition of *Everyday Mathematics* is developed over a period of years beginning with a research phase during which the authors review the most current research available related to how children learn. Initial drafts are extensively field tested, revised, and field tested again prior to publication.

After publication, the effectiveness of each edition is tested and proven by researchers at the University of Chicago School Mathematics Project (UCSMP) as well as independent researchers at other universities and institutions. The findings of these comprehensive research studies are further supported by data from individual schools and districts all over the country using *Everyday Mathematics*, data that consistently proves that the program helps children achieve more.



Engineered for Learning

Everyday Mathematics is a research-based curriculum design with instruction that is supported by research-based best practices.

Field Tested and Iteratively Improved

Extensive, rigorous field testing ensures that the curriculum is effective in both helping children learn and in helping teachers teach.

Verified Success

The effectiveness of the program is proven by independent research and in the success of districts using *Everyday Mathematics* across the country.

A Research-Based Approach to Improving Mathematics Education

Everyday Mathematics is developed and written by a group of education researchers at the University of Chicago School Mathematics Project (UCSMP) with the goal of helping elementary students acquire deeper conceptual understanding of mathematical concepts and greater mathematical fluency, helping them become life-long mathematical thinkers, problem solvers, collaborators.

Development of the first edition can be traced back to 1983 when, on the heels of a study that showed children in the United States lagging far behind their peers internationally in mathematics achievement, researchers at UCSMP began reviewing an exhaustive amount of existing research on children's mathematical thinking, curriculum, and instruction. Building on that knowledge, they conducted their own research, interviewing hundreds of children and studying instructional practices used in countries all over the world.

Based on what they learned, they established several guiding principles that informed the development of *Everyday Mathematics*, principles that have also been the foundation of every edition of the program.

The University of Chicago School Mathematics Project officially began in 1983. Since its inception it has been the largest university-based mathematics curriculum project in the United States.



Engineered for Learning

The Spiral Curricular Design

Spiraling refers to distributed practice as opposed to massed practice. Findings about the learning boost from spiraling are among the most robust in the learning sciences, applying across a wide range of content and for all ages from infants to adults. In fact, “Space learning over time” is the first recommendation in the U.S. Department of Education’s Institute of Educational Sciences (Pashler et al., 2007) practice guide. And in a recent review of the literature, Lisa Son and Dominic Simon write, “Both in the laboratory and the classroom, both in adults and in children, and in the cognitive and motor learning domains, spacing leads to better performance than massing” (2012).

Over a century of research has consistently proven

- Higher achievement on assessments
- Better, long-term mastery of math facts, skills, and concepts
- Faster identification of intervention needs

SELECTED ANNOTATED BIBLIOGRAPHY

Bjork, R.A. (1999). *Assessing our competence: Huristincs and illusions*. In D. Gopher & A. Koriat (Eds.), *Attention and performance XVII: Cognitive regulation of performance: Interaction of theory and application* (pp. 435-459). Cambridge, MA: MIT Press.

Summarizes the types of illusions of comprehension and competence and outlines the implications for real-world instruction.

Dempster, F.N. (1988). *The spacing effect: A case study in the failure to apply the results of psychological research*. *American Psychologist*, 43, 627-634.

Demonstrates the high potential for spaced learning to improve classroom learning and supports the application of spaced learning in classroom settings.

Pashler, H., Bain, P., Bottge, B., Graesser, A., Koedinger, K., McDaniel, M., & Metcalfe, J. (2007). *Organizing instruction and study to improve student learning* (NCER 2007-2004). Washington, DC: National Center for Education Research, Institute of Education Sciences, U.S. Department of Education. Retrieved from <http://ncer.ed.gov>.

Recommends the spacing of key course content as an overarching principle that teachers should attend to as they plan out sequences of instruction.

Rohrer, D. (2009). *The effects of spacing and mixing practice problems*. *Journal for Research in Mathematics Education*, 40, 4-17.

Explores research that demonstrates how dramatically test scores can be improved through spaced practice.

Son, L.K., & Simon, D.A. *Distributed learning: Data, metacognition, and educational implications*. *Educational Psychology Review* (2012): 1-21.

Discusses recommendations regarding how and why spacing strategies might be encouraged in real-world learning.

* See appendix for a more complete bibliography.

Engineered for Learning

Raising Achievement by Raising Expectations

Children begin school with a great deal of knowledge and intuition on which to build: making use of this knowledge helps children achieve greater conceptual understanding.

Numerous studies confirm that young children, regardless of socio-economic background, possess considerable informal mathematical knowledge, which most curricula fail to use (Riley, Greeno, & Heller, 1983; Carpenter & Moser, 1984; Hiebert, 1984; Cobb, 1985; Baroody & Ginsburg, 1986; Bell & Bell, 1988; Resnick, Lesgold, & Bill, 1990; Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993).

Researchers have also found that children have much richer and more active mathematical minds than had been suspected (Gelman and Gallistel, 1978; Gelman, 1982; Resnick, 1983; Fuson & Hall, 1983; Gelman, Meck, & Merkin, 1986). These studies show that young children are capable of absorbing a great deal of new material, sometimes more rapidly than adults.

In addition, volumes of research have shown the positive impact of learning in a social context. For example, a problem that seems beyond the capabilities of a child working alone can often be solved when appropriate manipulatives are available, and children are allowed to interact with each other.

Through the use of manipulatives and small-group work, combined with instruction that builds on children's experiences and makes connections between those experiences and the discipline of mathematics, *Everyday Mathematics* helps teachers bring focus and coherence to their students' learning, while supporting the development of true, long-term mastery of mathematical topics.

Most kindergarten children are capable of solving a wide range of simple addition and subtraction story problems by their own methods.

Riley, Greeno, & Heller, 1983;
Carpenter and Moser, 1984

Research on children's informal solution methods revealed a typical developmental progression from simple counting of objects, to use of more sophisticated counting strategies and relationships, to derived fact strategies, to use of arithmetic facts and number relationships.

Bergeron & Herscovics, 1990; Fuson, 1992

Implementing Classroom Practices that Deliver Balanced Rigor

One of the perennial arguments in education is between those who want students to develop skill in carrying out procedures and those who want students to understand the concepts behind why those procedures work. In reality, this is a false choice. Children with weak conceptual understandings are hindered in their skill development, and children with weak skills are handicapped as they work towards higher levels of conceptual understanding (Carpenter, 1986).

Additional research has also pointed out the unfortunate outcomes when a proper balance between meaning and skill is not maintained (Skemp, 1978; Baroody & Ginsburg, 1986; Resnick, 1987b).

One approach to answering the question of how to best deliver balanced instruction is to look at curricula used in nations that outperform U.S. students in terms of both skills development and conceptual understanding. Reviews of these programs found that they employ more child-centered, problem-solving approaches to instruction in mathematics when compared to most U.S. programs. (Stevenson & Stigler, 1992; Stigler & Perry, 1988).



“Children who could not make sense of the expression ‘12 divided by 3’ could easily respond correctly to the request, ‘Share these blocks among you, me, and my friend.’”

Bell and Bell, 1988

SELECTED ANNOTATED BIBLIOGRAPHY

Balfanz, R. (1990). Elementary school quality, the mathematics curriculum and the role of local knowledge. *International Review of Education* 36(1): 43-56.

Argues that a key means by which elementary school quality can be improved is to begin with the knowledge students develop on their own and transform it through pedagogic and curricular intervention into a set of portable intellectual skills.

Bergeron, J.C., & Herscovics, N., (1990). Psychological aspects of learning early mathematics. In P. Neshet & J. Kilpatrick (Eds.), *Mathematics and Cognition: A Research Synthesis by the International Group for the Psychology of Mathematics Education* (pp. 31-52). Cambridge, England, Cambridge University Press.

Through case studies, Bergeron & Herscovics conclude: (1) mathematics is a thinking process, not a mastery of skills; (2) children possess a greater knowledge of mathematics than previously accepted; and (3) if teachers realize that children are capable of more challenging mathematics, the instructional focus will change from end results to thought processes.

Bell, M. S. (1972). Mathematical uses and models in our everyday world. *Studies in mathematics, volume XX*. Stanford: School Mathematics Study Group, 1972. (ERIC ED 143-557)

Presents a comprehensive collection of mathematical problems that highlight the applications of mathematics in real-life situations.

Isaacs, A. & Carroll, W.M. (1999). Strategies for basic facts instruction. *Teaching Children Mathematics*, 5 (9), pp. 508-515.

Describes a strategies approach to basic addition and subtraction-facts instruction, and discusses assessment techniques and a rationale for the approach.

* See appendix for a more complete bibliography.

Field Tested and Iteratively Improved

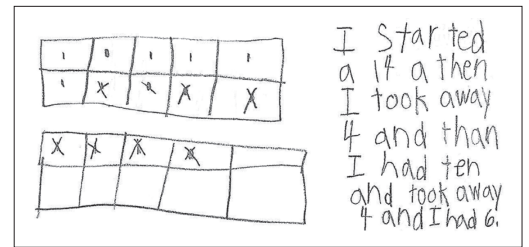
Extensive Field Testing

Prior to publication, each edition of *Everyday Mathematics* is rigorously field tested in classrooms across the country. The authors conduct formative assessments of their work, using rigorous and systematic procedures for gathering and analyzing implementation and achievement data. In addition, they interview and observe teachers and students using the material in hundreds of real classrooms across the country.

Revisions are made based on the empirical findings of this research and then re-tested in the field. This iterative development process, which is unique to *Everyday Mathematics*, helps ensure that every lesson supports how children learn and that the lessons work in actual classrooms.

This process has been enhanced by the introduction of the *Everyday Mathematics* ConnectED Teacher Center and Student Learning Center, giving the authors even more opportunities for field testing and iterative improvement through extensive, ongoing testing in digital classrooms all over the country.

- During initial development, *Everyday Mathematics* was field tested at each grade for a full year.
- Over 800 students participated in field testing 170 lessons in *Everyday Mathematics 4*.
- Open Response problems as well as Open Response and Reengagement Lessons were field tested by more than 1400 students.



① Show at least four possible coin combinations Carlos could use to pay for the milk. Use N , D , and Q to record your answers.

① QQ DDN ② DDDDD NNNNN

③ QQNNNNN ④ Q DDDDD

② Pick one of your coin combinations and show or explain how you know it totals exactly 75¢.

I know two quarters makes 50¢ and I added two dimes and that made 70¢ and then I added a Nickele and that made 75¢.

Field testing not only ensures that *Everyday Mathematics* works, it also allows the authors to gather examples of actual student work and publish them in the *Teacher's Lesson Guide* to help teachers assess their own students' work.

Everyday Mathematics Research and Development

Ample time is given to research, test, and write each edition prior to publication.

1983-1988

Survey of Existing Research into How Children Learn, Translation and Survey of International Curricula, Research into Effective Curricular Design

2001-2003

Pre-K Curriculum Researched, Written, Field Tested, Revised, and Published

First Edition

1989-1996

Focusing on One Grade a Year, Each Grade is Written, Field Tested, Revised, and Published as the **First Edition**

Second Edition

1997-2000

Second Edition, Grades K-3 Researched, Written, Field Tested, and Revised

2001

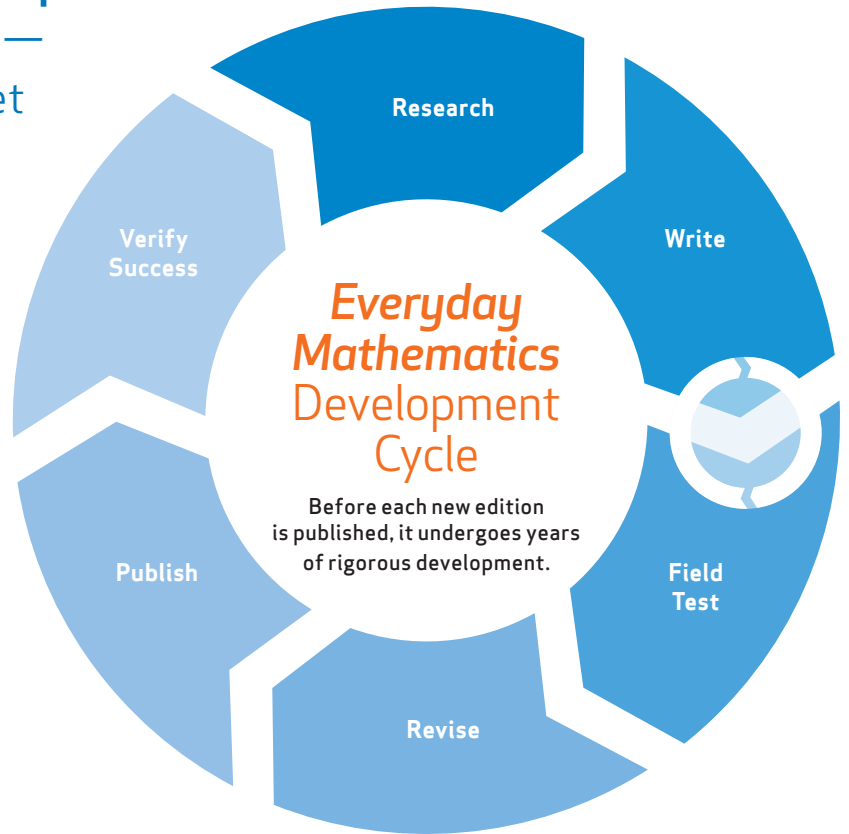
Second Edition Published

1999-2001

Second Edition, Grades 4-6 Researched, Written, Field Tested, and Revised

“We love figuring out how to use the latest findings from research in the learning sciences to **build tools that help kids learn mathematics**. And then—what’s even more fun—we get to study how teachers in schools all across the country use those tools and so we can revise what we’ve created **based on what really works.**”

Andy Isaacs, CEMSE Director,
Director of *Everyday Mathematics* Revisions

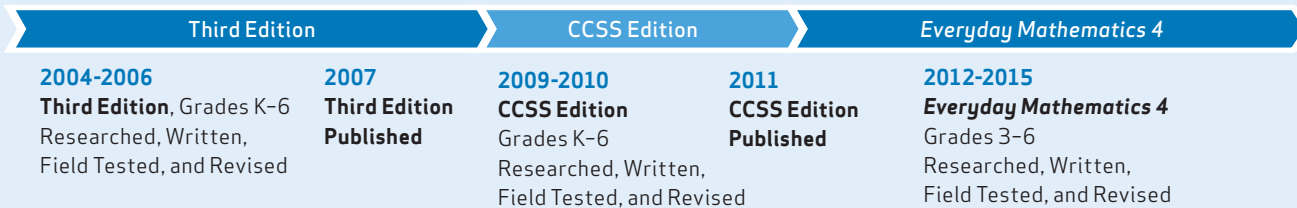


2011-2014

Everyday Mathematics 4
Grades K-2
Researched, Written,
Field Tested, and Revised

2015

Everyday Mathematics 4
Published



Verified Success

After the materials are final or near-final, summative evaluations are conducted that demonstrate the achievement differences that educators using *Everyday Mathematics* can expect.

These studies are led by independent researchers, researchers at UCSMP, and by schools and districts using the program. Overall, the studies have consistently shown that the program is effective in real classrooms with real students.

Learner Verification and Evaluation Studies

THE NORTHWESTERN LONGITUDINAL STUDY

Everyday Mathematics was the focus of a five-year longitudinal curriculum study designed and conducted by researchers at Northwestern University. The study included student and teacher interviews, classroom observations, written tests, collected artifacts, and surveys. This longitudinal study used a variety of instruments and observational methods. Items on written tests were drawn from the National Assessment of Educational Progress (NAEP), from international studies of mathematics achievement, and from the research literature.

Researchers using the data and findings of the Northwestern study have found that *Everyday Mathematics* students constantly outperform comparison students (Carroll 2000a, Fuson 2000).

TRI-STATE ACHIEVEMENT STUDY

The ARC Center, located at the Consortium for Mathematics and its Applications (COMAP), completed a study that compared the effects of standards-based mathematics programs on student performance with state-mandated standardized tests in Massachusetts, Illinois, and Washington. The National Science Foundation funded this study and its report.

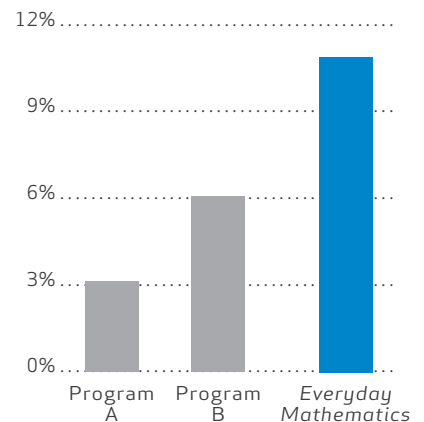
The reports' findings are based on the records of over 78,000 students: 39,701 who had used the *Everyday Mathematics* curriculum for at least two years, and 38,481 students from comparison schools. The students were carefully matched by reading level, socioeconomic status, and other variables.

Results showed that the average scores of students in the *Everyday Mathematics* schools were consistently higher than the average scores of students in the comparison schools. The results hold across different state-mandated tests and across topics ranging from computation, measurement, and geometry to algebra, problem-solving, and making connections.

WHAT WORKS CLEARINGHOUSE™ IMPROVEMENT INDEX

The U.S. Department of Education What Works Clearinghouse™ recognizes *Everyday Mathematics* as the most effective core elementary mathematics program in the country.

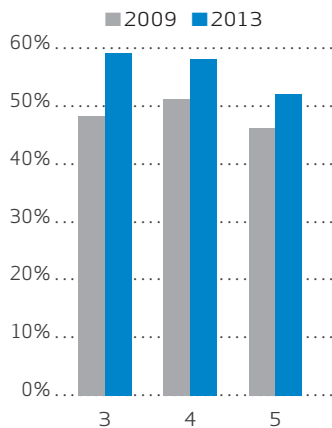
Expected Percentile Gain for the Average Student using *Everyday Mathematics* versus other programs.



High Achievement in Denver

Denver's diverse student population made significant gains in all grades on the Transitional Colorado Assessment Program with the help of *Everyday Mathematics*. Since implementing new standards in 2009, the percentage of students performing at or above proficiency has increased more than 10% in each grade.

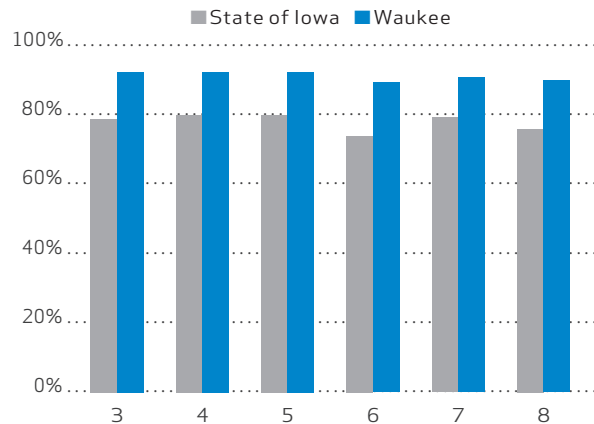
Denver Public School District Transitional Colorado Assessment Program—Mathematics Percentage Advanced or Proficient, Grades 3-5



Waukee Community School District Sustains Success

Long-time users of *Everyday Mathematics*, elementary students in Waukee, IA consistently demonstrate higher proficiency than their peers across the state, gains that remain consistent through middle school.

2013 Adequate Yearly Progress in Mathematics—Percent Proficient, Grades 3-8

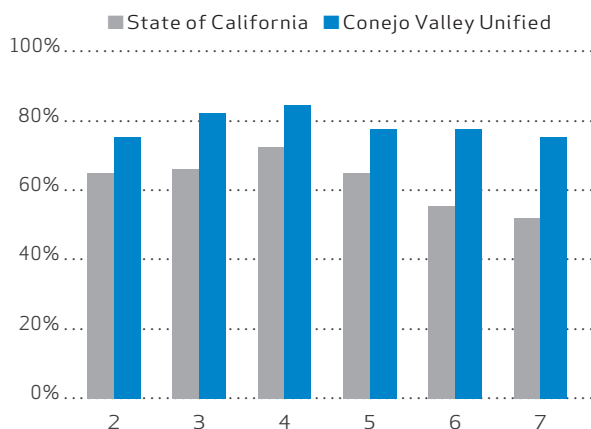


Conejo Valley Outperforms State of California

Long-time users of *Everyday Mathematics*, Conejo Valley consistently outperforms the state of California on the California Standards Test in Mathematics.

In addition, scores at the state level decline sharply in upper grades, but scores in Conejo Valley remain high.

Conejo Valley Unified School District 2013 California Standards Test in Mathematics—Percentage Proficient or Advanced, Grades 2-7

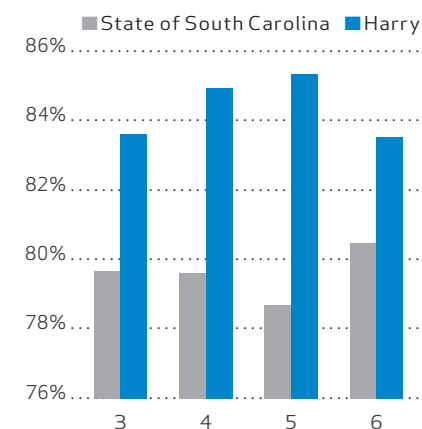


Horry County, South Carolina Exceeds Standards

Students in Horry County consistently outperform their peers across the state, both in overall mathematics proficiency and in proficiency with each state standard.

For example, for domain 2, Number and Operations, the students of Horry County rate at an average of 4 points higher than the entire state in each grade.

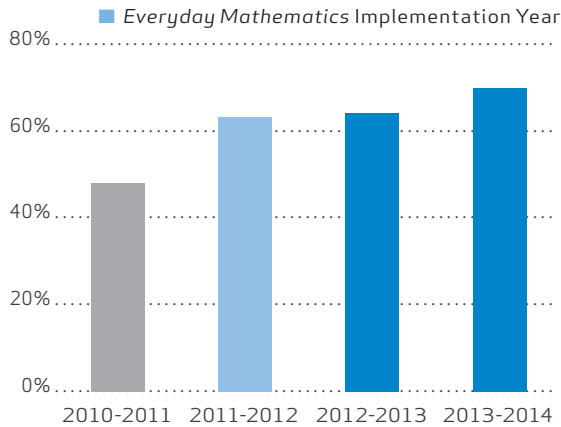
Palmetto Assessment of State Standards 2014 Math Standard 2 (varies for each grade)—Percent At or Above Proficient, Grades 3-6



Implementation Bump in Murfreesboro

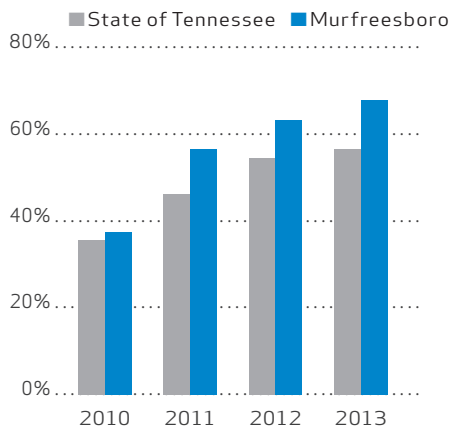
One year after implementing *Everyday Mathematics*, scores in Murfreesboro City School District saw an increase of 15% on the Grade 3 Tennessee Comprehensive Assessment Program (TCAP) during the **first year** (2011) of its *Everyday Mathematics* implementation.

Murfreesboro City School District Tennessee Comprehensive Assessment Program (TCAP)—Mathematics Grade 3, Percent Advanced or Proficient



In addition, 3rd graders using *Everyday Mathematics* from the implementation year forward, continued to widen the performance gap when measured against with their peers across the state as they moved into Grade 5.

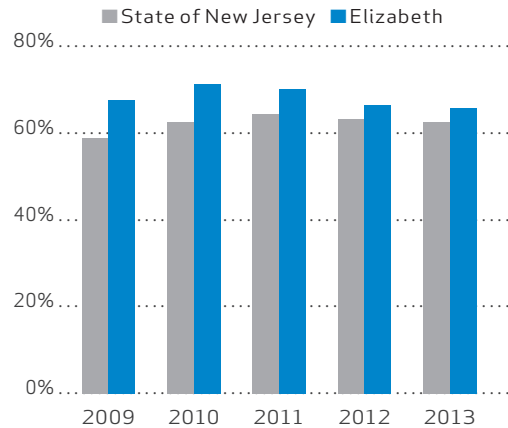
Murfreesboro City School District Tennessee Comprehensive Assessment Program (TCAP)—Mathematics Grade 5, Percent Advanced or Proficient



New Jersey's Life-Long Mathematical Thinkers

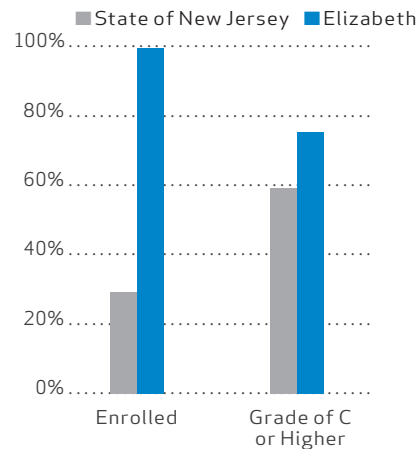
Students in the Elizabeth Public Schools, where over 85% of students receive free or reduced lunches, consistently outperform districts with similar demographics across the state.

Elizabeth School District New Jersey Assessment of Skills and Knowledge (NJ ASK)—Mathematics Percentage At or Above Proficient—Economically Disadvantaged Students, Grade 3



In addition, 100% of elementary students in Elizabeth go on to take a high-school algebra course, compared to less than 30% for the state. And, 75% of them score a grade of "C" or better.

Algebra Enrollment and End of Course Grades 2012-2013 School Year



For more *Everyday Mathematics* Success Stories, visit everydaymath.com

How Children Learn

Everyday Mathematics 4 is built on a foundation of decades of research into how children learn and has been field tested with teachers and children in real classrooms to ensure it will lead to successful outcomes for your children.

This development approach is unique to *Everyday Mathematics* and is made possible by the generosity of the teachers and administrators all over the country who have opened their classrooms for observations and field tests for over 30 years. The data gathered through these meetings, surveys, classroom observations, and interviews has been an integral part of the development of every component of every edition of the program.

The research and testing required mean that each edition of *Everyday Mathematics* takes years to develop, but it makes the program the most effective elementary mathematics program available, a fact that has been consistently proven by research, but perhaps more importantly, can also be seen in the increased achievement of students all over the country.

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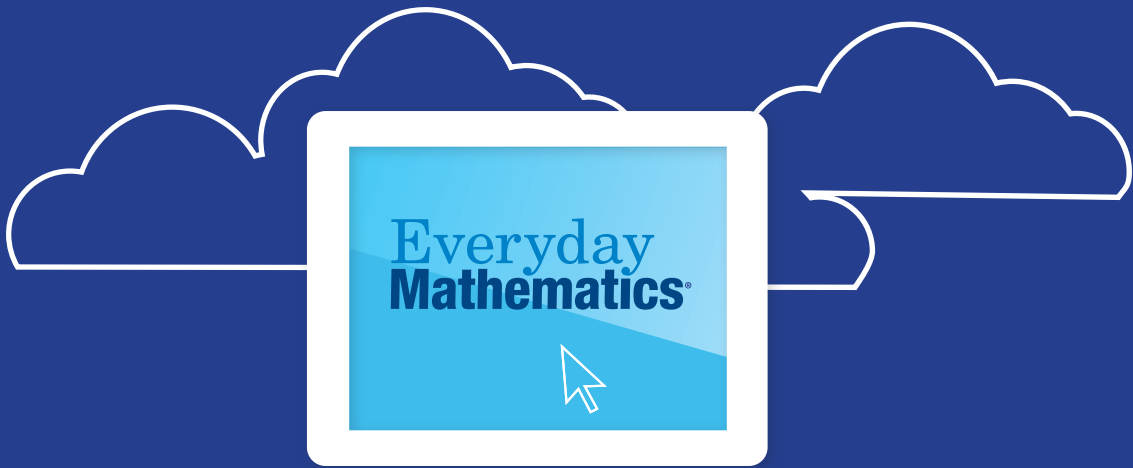
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